MATH 590: QUIZ 3

Name:

Throughout V and W will denote a vector space over F, where $F = \mathbb{R}$ or $F = \mathbb{C}$.

1. Define what it means to say that V has dimension n. (2 points)

Solution. V has dimension n if it has a basis consisting of n elements.

2. Define what it means for $T : V \to W$ to be a linear transformation. (2 points)

Solution. T is a linear transformation if $T(v_1 + v_2) = T(v_1) + T(v_2)$ and $T(\lambda v) = \lambda T(v)$, for all $v_1, v_2, v \in V$ and $\lambda \in F$.

3. Find a basis for the vector space V space of real symmetric 2×2 matrices. No need to justify your answer. (3 points)

Solution. A typical symmetric matrix is of the form $\begin{pmatrix} a & b \\ b & c \end{pmatrix} = a \cdot \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + b \cdot \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + c \cdot \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ showing that $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$ span V. It is easy to see that these matrices are linearly independent over $\mathbb{R}$, and thus form a basis for V.

4. Show that $T : \mathbb{R}^3 \to \mathbb{R}$ defined by $T((a, b, c)) = 3a + 2b + c$ is a linear transformation. (3 points)

Solution. Suppose $v_1 = (a, b, c)$ and $v_2 = (d, e, f)$. Then

$$\begin{aligned} T(v_1 + v_2) = T((a, b, c) + (d, e, f)) &= T((a + d, b + e, c + f)) \\ &= 3(a + d) + 2(b + e) + (c + f) \\ &= (3a + 2b + c) + (3d + 2e + f) = T(v_1) + T(v_2). \end{aligned}$$

And:

$$T(\lambda v_1) = T((\lambda a, \lambda b, \lambda c)) = 3(\lambda a) + 2(\lambda b) + (\lambda c) = \lambda(3a + 2b + c) = \lambda T(v_1).$$